



**Answer the following questions:**

**Question (1)**

(a) Test the series  $\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$  for convergence and find the interval of convergence

for the power series  $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 2^n}$

(b) Given  $w = \tan^{-1}(x^3 + y^3)$  show that  $x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} = 3 \sin w \cos w$

(c) Find the local extrema of the function  $f(x, y) = -x^2 - 4x - y^2 + 2y - 1$ .

**Question (2)**

(a) For any scalar function  $\varphi(x, y, z)$  show that  $\text{curl grad } \varphi = 0$

(b) Find the area enclosed by the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ .

(c) Find the area bounded by the curves  $xy = 4, xy = 8, xy^3 = 5, xy^3 = 15$

**Question (3)**

Solve the following differential equations

(a)  $(xy - x^2)dy - y^2 dx = 0$

(b)  $(xy^3 - 1)dx - x^2 y^2 dy = 0$

(c)  $y'' - 6y' + 13y = 8e^{3x} \sin 2x$

**Question (4)**

(a) Find the general solution for Euler equation  $x^2 y'' - xy' + 2y = x \ln x$

(b) Use variation of parameter to solve  $y'' + n^2 y = \sec nx$ .

(c) Solve  $xy'' - (2x+1)y' + (x+1)y = 0$  given that  $y = e^x$  one solution.

### Question (5)

(a) For the vector field  $\vec{F} = (4xy - 3x^2z^2)\vec{i} + 2x^2z\vec{j} - 2x^3z\vec{k}$  prove that

$\oint_C \vec{F} \cdot d\vec{r}$  independent to any path through two any points in domain  $\vec{F}$  and find

the scalar potential function  $\varphi$  which satisfy  $\vec{F} = \nabla\varphi$ .

(b) Evaluate  $\iint_S \vec{F} \cdot \vec{n} ds$  where  $\vec{F} = 2yx\vec{i} + yz^2\vec{j} + xz\vec{k}$  and  $S$  is the surface of parallelogram bounded by  $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3$

(c) Apply Stock and Green theorem to evaluate  $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS$  where

$\vec{F} = (x^2 + y - 4)\vec{i} + (3xy)\vec{j} + (2xz + z^2)\vec{k}$  and  $S$  is the surface bounded by the paraboloid  $z = 4 - (x^2 + y^2), z \geq 0$ .